



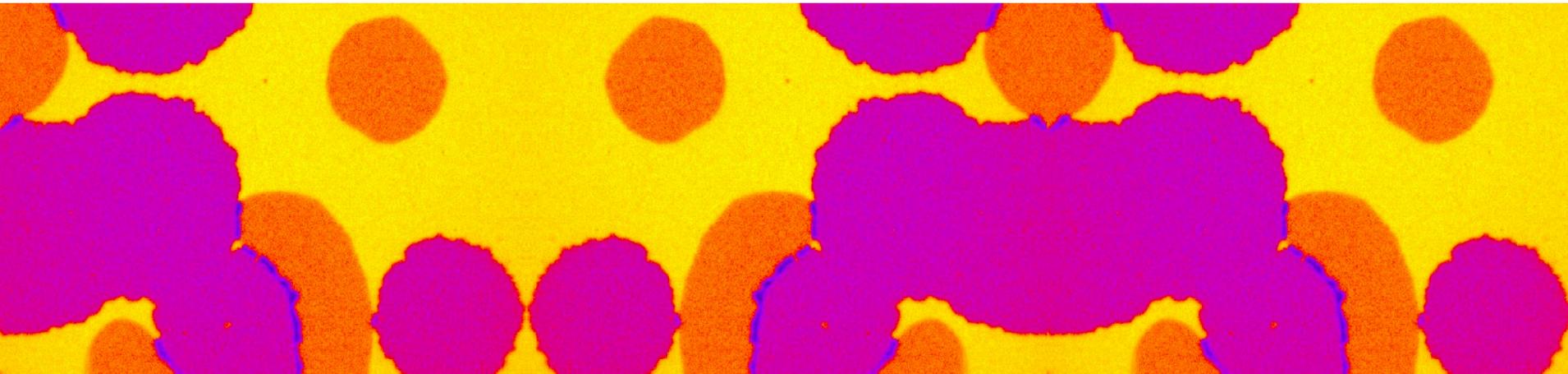
University  
of Glasgow



# Magnetism and Magnetic Switching

*Robert Stamps*

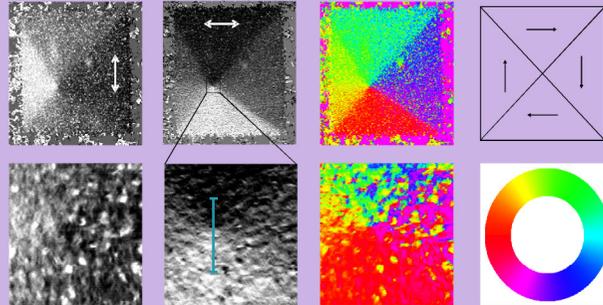
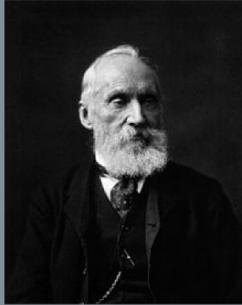
*SUPA-School of Physics and Astronomy  
University of Glasgow*



# Kelvin Nanocharacterisation Centre

# Materials & Condensed Matter Physics

William  
Thomson  
(Kelvin)



Standard mode: < 100 pm  
Lorentz mode: < 2 nm



James  
Watt



William  
Rankine



Magnetic Nanostructures  
Theory and Modelling  
Functional Materials  
Electron Microscopy



# A story from modern magnetism: **The Incredible Shrinking Disk**

Instead of this:  
(1980)



# A story from modern magnetism: **The Incredible Shrinking Disk**

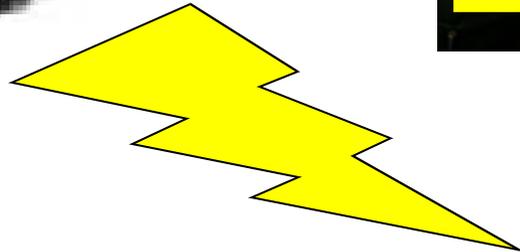
We have this:  
( $10^3$  greater capacity)



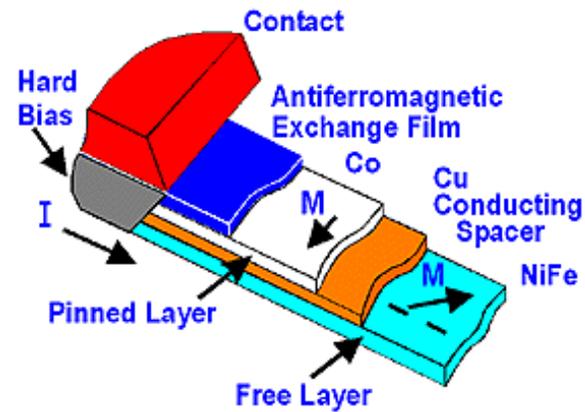
# Size and Sensitivity



*Nobel Prize 2007*

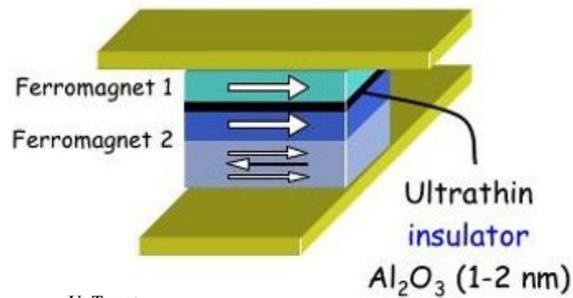


The physics of GMR **sensors** allow 10 to 100 times sensitivity over previous sensors. Greater sensitivity means ability to read data at higher densities.



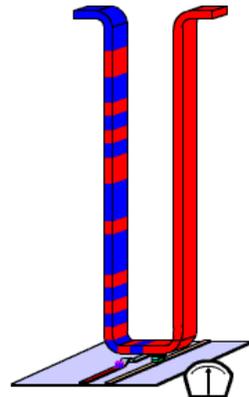
# Devices for Tomorrow

## Magnetic RAM



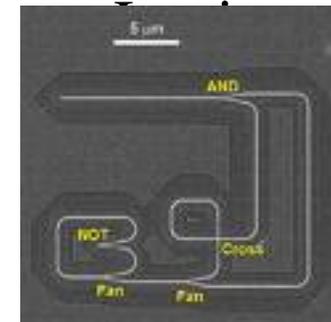
U. Twente

## Racetrack Memory

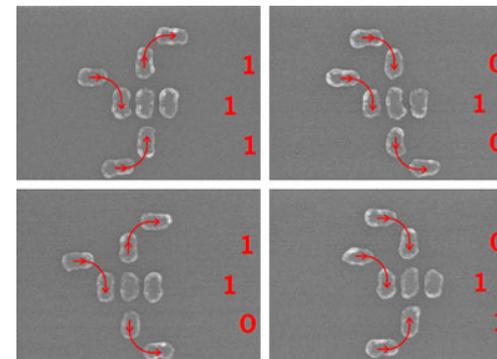


IBM Almaden

## Nanomagnetic



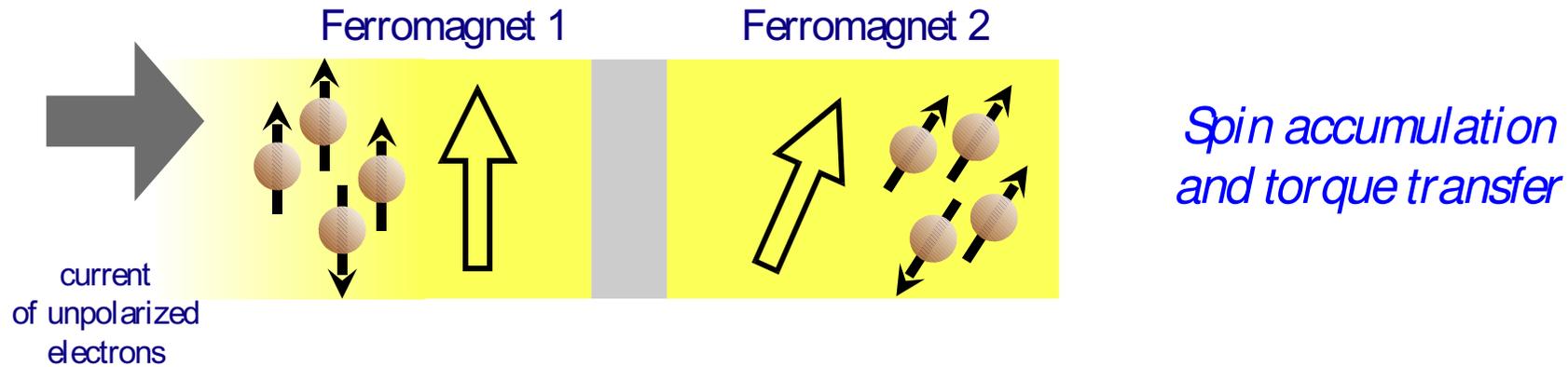
U. Sheffield



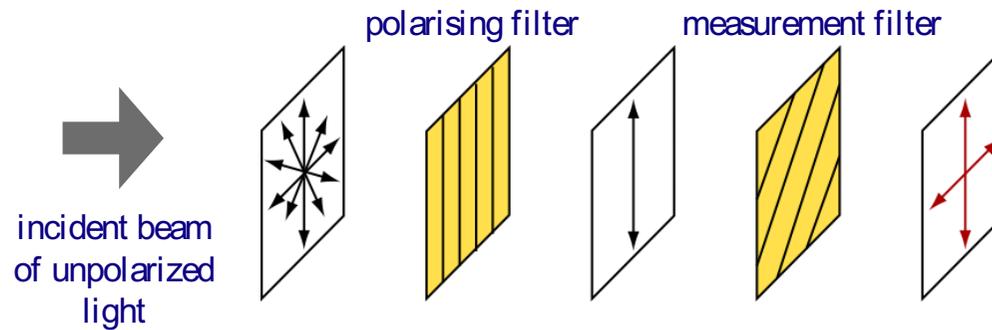
Source: University of Notre Dame

Reversal involves **precessional magnetic rotation**, domain wall formation and **domain wall motion**

# Spin Torque Transfer & Spin Currents

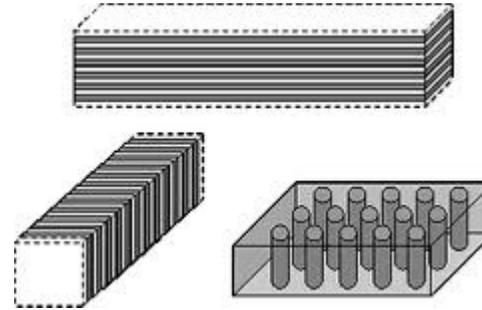
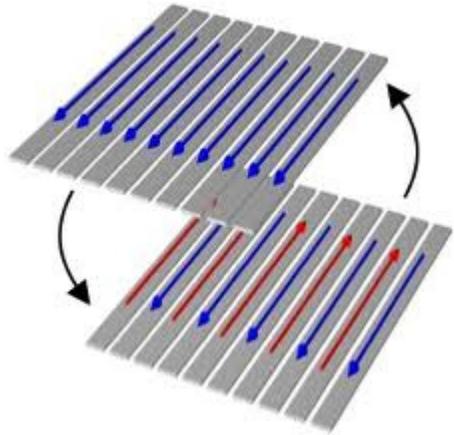


## Optical analogy:

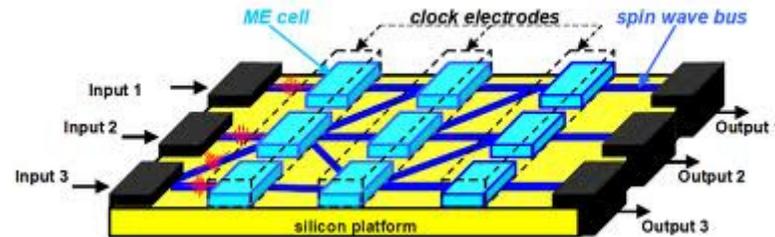
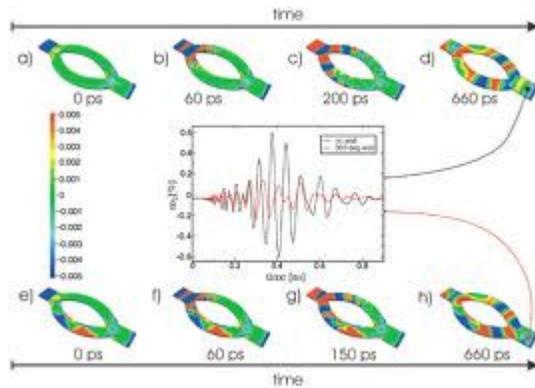


# Magnonics

**Control GHz properties** with patterned magnetic films:



**Spinwave based logic:**

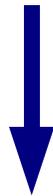


# Outline:

- **Starting points: Magnetic moment and exchange**
  - Electrostatics and exchange energy
  - Magnetic ordering and mechanisms for exchange
- **Magnetisation dynamics**
  - Torque equations and effective fields
  - Precessional dynamics
  - Switching
- **Spin waves**
  - Correlated precession

# Starting Point 1: Magnetic Moment

Spin and orbital **angular momentum**:

  $\hbar \mathbf{J} = \hbar (\mathbf{S} + \mathbf{L})$

  $\boldsymbol{\mu} = \gamma (\hbar \mathbf{J})$

*Gyromagnetic ratio  $\gamma$ :*

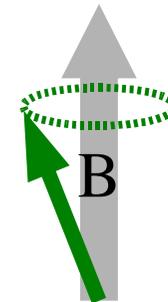
$$\gamma = -g \frac{\mu_B}{\hbar}$$

Energy and **precession**:

$$U = -\boldsymbol{\mu} \cdot \mathbf{B}$$

$$\boldsymbol{\Gamma} = \frac{d}{dt} \mathbf{L} = \boldsymbol{\mu} \times \mathbf{B}$$

$$\mathbf{L} = \mathbf{L}_0 + l e^{-i\omega_L t}$$

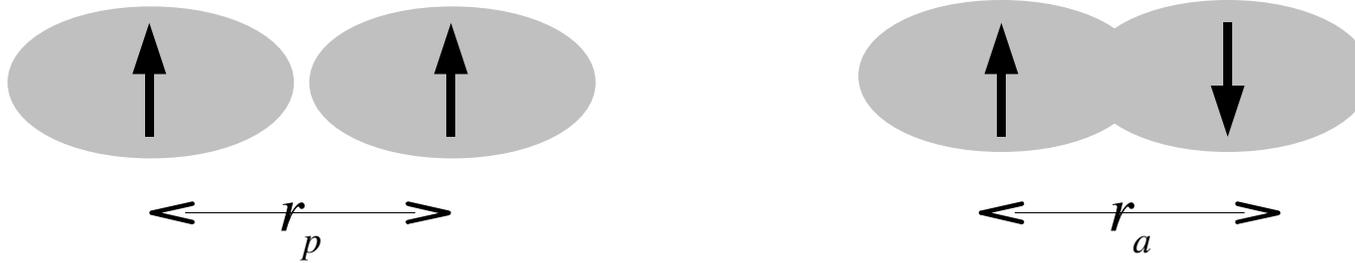


*Larmor precession*

$$\omega_L = \frac{\mu_B}{\hbar} B$$

# Concept: Exchange Energy

Pauli exclusion **separates** like spins:



Can be **energetically favourable**: *suppose* parallel alignment results in a small change in average separation. Then *if*:

$$\left. \begin{array}{l} \langle r_a \rangle \sim 0.3 \text{ nm} \quad \Rightarrow \quad \frac{e^2}{r_a} \sim 4.8 \text{ eV} \\ \langle r_p \rangle \sim 0.31 \text{ nm} \quad \Rightarrow \quad \frac{e^2}{r_p} \sim 4.75 \text{ eV} \end{array} \right\} E_{\uparrow\uparrow} - E_{\uparrow\downarrow} = 0.05 \text{ eV} (580 \text{ K})$$

... equivalent field:  $\frac{E_{\uparrow\uparrow} - E_{\uparrow\downarrow}}{\mu_B} = 870 \text{ T}$

# Example: Direct Exchange

Electrons on two **neighbouring atoms**:

$$H = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + \frac{e^2}{|\mathbf{R}_a - \mathbf{R}_b|} - \underbrace{\left( \frac{e^2}{|\mathbf{r}_1 - \mathbf{r}_a|} + \frac{e^2}{|\mathbf{r}_2 - \mathbf{r}_b|} \right)}_{\text{intra-atomic}} + \frac{e^2}{|\mathbf{r}_1 - \mathbf{r}_2|}$$

*core-core*
*inter-atomic*

Assume know solutions for **isolated orbitals** with energy  $E$  on atoms  $a$  and  $b$ .

Solve two electron problem with **combination of product orbitals**

$$\left. \begin{aligned} \Psi_I &= \phi_a(\mathbf{r}_1) \phi_b(\mathbf{r}_2) \\ \Psi_{II} &= \phi_a(\mathbf{r}_2) \phi_b(\mathbf{r}_1) \end{aligned} \right\} \psi = c_I \Psi_I + c_{II} \Psi_{II}$$

# Example: Direct Exchange

Solution involves several **overlap integrals**:

$$V = \iint d\mathbf{r}_1 d\mathbf{r}_2 |\Psi_{I,II}^2| e^2 \left( \frac{1}{R_{ab}} + \frac{1}{r_{12}} - \frac{1}{r_{ab}} - \frac{1}{r_{1a}} - \frac{1}{r_{2b}} \right)$$

$$U = \iint d\mathbf{r}_1 d\mathbf{r}_2 \Psi_I^* \Psi_{II} e^2 \left( \frac{1}{R_{ab}} + \frac{1}{r_{12}} - \frac{1}{r_{ab}} - \frac{1}{r_{1a}} - \frac{1}{r_{2b}} \right)$$

$$l = \int d\mathbf{r} \phi_a^*(\mathbf{r}) \phi_b(\mathbf{r})$$

Two electron **wavefunctions**:

*space symmetric*

$$c_I = c_{II}$$

$$E_+ = 2E + \frac{V+U}{1+l^2}$$

*space anti-symmetric*

$$c_I = -c_{II}$$

$$E_- = 2E + \frac{V-U}{1-l^2}$$

# Example: Direct Exchange

For **Pauli exclusion** require

$$\psi_s \sim (\text{space symmetric})(\text{spin antisymmetric})$$

$$\psi_a \sim (\text{space antisymmetric})(\text{spin symmetric})$$

$$\psi_s \rightarrow \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$$

*spin 0*

$$\psi_a \rightarrow (\downarrow\downarrow) \quad \frac{1}{\sqrt{2}}(\uparrow\uparrow + \downarrow\downarrow) \quad (\uparrow\uparrow)$$

*spin -1, 0, 1*

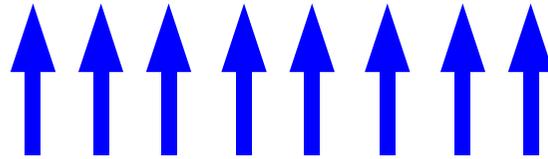
**Energy difference** determines whether spins prefer **parallel or antiparallel** alignment:

$$J = E_- - E_+ \sim U l^2 - V$$

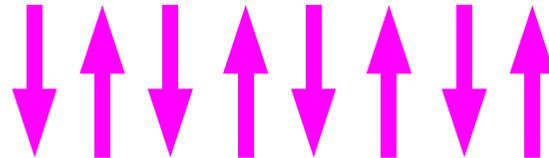
Heitler & London (1927)

# Examples of Magnetic Ordering

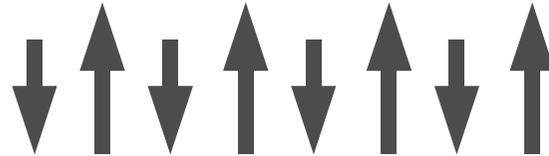
Ferromagnetic:



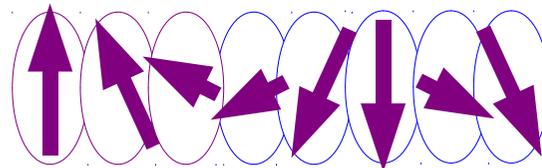
Antiferromagnetic:



Ferrimagnetic:

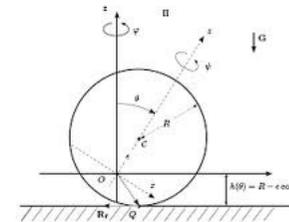
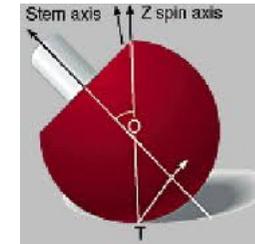
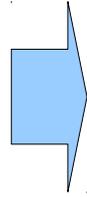


Helical:



# Summary I

Bohr and Pauli Study  
Angular Momentum



**Angular momentum & magnetic moment:**

- Defines energy, torque and precession

**Exchange: electrostatic repulsion & Pauli exclusion:**

- Determines long range order and phase transitions

# Starting Point 2: Phenomenology

Relevant energy scales (P. W. Anderson, 1953):

1 – 10 eV

Atomic Coulomb integrals  
Hund's rule exchange energy  
Electronic band widths  
Energy/state at  $\epsilon_f$

*ordering*

0.1 – 1.0 eV

Exchange energy splitting

$10^{-2}$  –  $10^{-1}$  eV

Spin-orbit coupling

*spin waves*

$10^{-4}$  eV

Magnetic spin-spin coupling  
Interaction of a spin with 10 kG field

$10^{-6}$  –  $10^{-5}$  eV

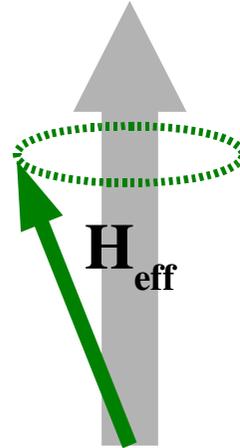
Hyperfine electron-nuclear coupling

# Dynamics & Effective Field

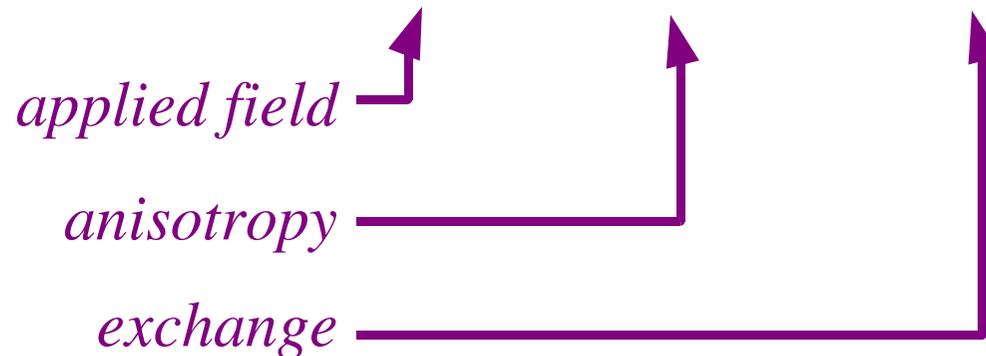
Magnetic parameters describe energy & torques:

$$E = -\mathbf{m}(\mathbf{r}, t) \cdot \mathbf{H}_{\text{eff}}(\mathbf{r}, t)$$

$$\frac{\partial}{\partial t} \mathbf{m}(\mathbf{r}) = -\gamma \mathbf{m}(\mathbf{r}) \times \mathbf{H}_{\text{eff}}$$



$$\mathbf{H}_{\text{eff}} = \mathbf{H}_a + \mathbf{K}(\mathbf{m}(\mathbf{r})) + A \nabla^2 \mathbf{m}(\mathbf{r}) - \mathbf{h}_{\text{dip}}$$



# Magnetisation & Exchange Parameters

**Basic idea:** define magnetisation density

$$\hat{\mathbf{M}}(\mathbf{r}) = g \mu_B \sum_j \delta(\mathbf{r} - \mathbf{r}_j) \hat{\boldsymbol{\sigma}}_j$$

**Exchange energy must be compatible with symmetry of local atomic environment:**

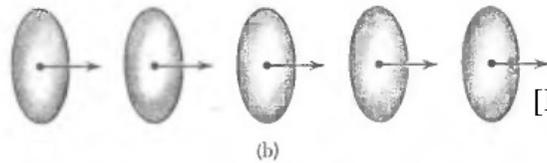
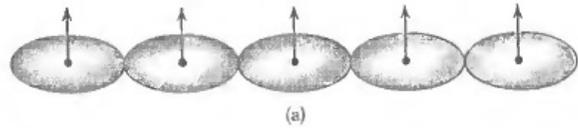
$$E_{ex} = \sum_{\alpha kl} C_{kl} \frac{\partial m_{\alpha}(\mathbf{r})}{\partial r_k} \frac{\partial m_{\alpha}(\mathbf{r})}{\partial r_l}$$

**Example: isotropic medium**

$$E_{ex} = A [(\nabla m_x)^2 + (\nabla m_y)^2 + (\nabla m_z)^2]$$

# Magnetic Anisotropy Parameter

**Local atomic environment affects spin orientation:**



[Kittel, Introduction to Solid State]

*Spin orbit interaction  
and crystal field  
effects*

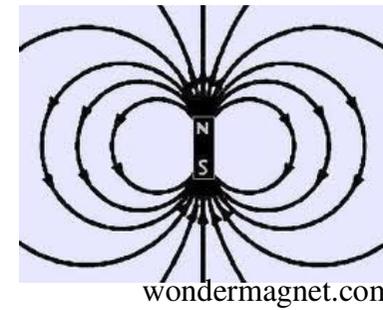
**Anisotropy energy and symmetries:**

• Uniaxial:  $E_{ani}(m_z) = E_{ani}(-m_z)$   $\Rightarrow E_{ani} = -K_u^{(1)} m_z^2 - K_u^{(2)} m_z^4 + \dots$

• Cubic:  $E_{ani}(m_x, m_y, m_z) = E_{ani}(-m_x, m_y, m_z)$ , etc.

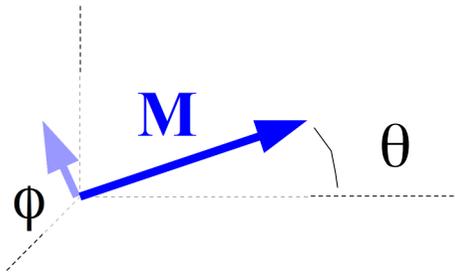
$\Rightarrow E_{ani} = K_4 (m_x^2 m_y^2 + m_x^2 m_z^2 + m_y^2 m_z^2) + \dots$

# Dipolar Interactions



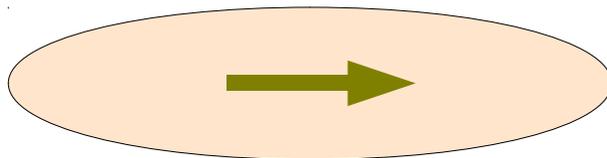
Magnetic moments are **point dipoles**:

$$\mathbf{h}_{dip}(\mathbf{r}_{ij}) = \frac{1}{2} g^2 \mu_B^2 \sum_{ij} \left( \frac{\mathbf{m}(r_i) \cdot \mathbf{m}(r_j)}{r_{ij}^3} - 3 \frac{[\mathbf{r}_{ij} \cdot \mathbf{m}(r_i)][\mathbf{r}_{ij} \cdot \mathbf{m}(r_j)]}{r_{ij}^5} \right)$$

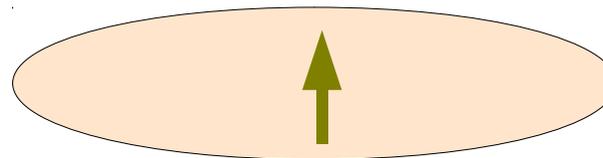


**All moments interact throughout sample. Sample shape contributes to anisotropy. For an ellipsoid:**

$$E_{ani} = \frac{M^2 V}{2\mu_o} (N_x \sin^2 \theta \cos^2 \phi + N_y \sin^2 \theta \sin^2 \phi + N_z \cos^2 \theta)$$



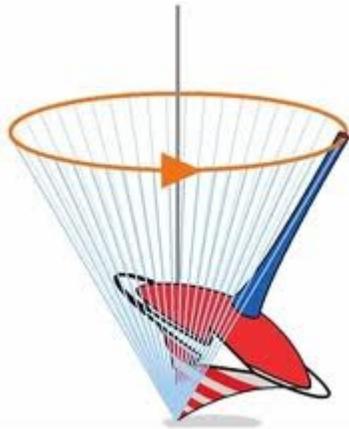
*Easy direction*



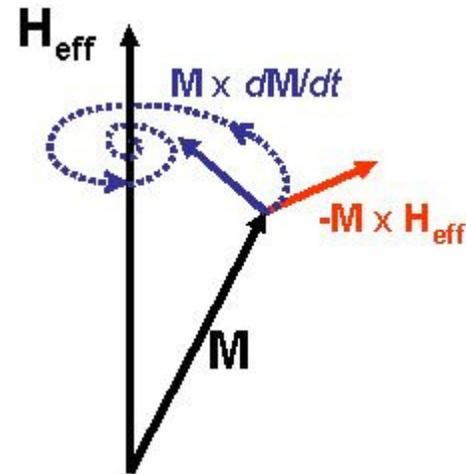
*Hard direction*

# Dissipation

**Damping:** additional torques



www.medical.siemens.com

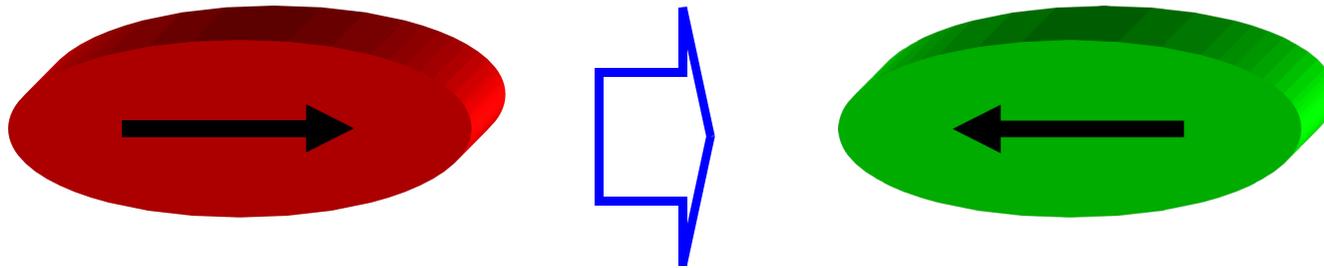


www.ptb.de

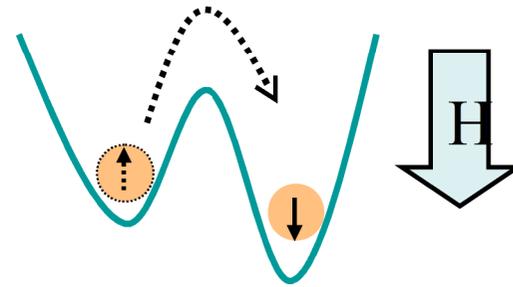
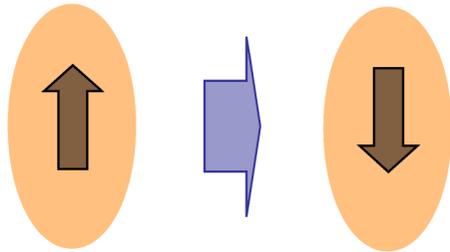
**Gilbert damping:** Rayleigh dissipation form

$$\frac{\partial}{\partial t} \mathbf{m}(\mathbf{r}) = \gamma \mathbf{m}(\mathbf{r}) \times \mathbf{H}_{\text{eff}} - \alpha \mathbf{m}(\mathbf{r}) \times \frac{\partial \mathbf{m}(\mathbf{r})}{\partial t}$$

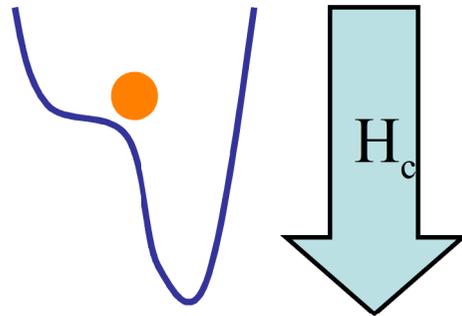
# Magnetic Switching



# Switching of Single Domain Particles

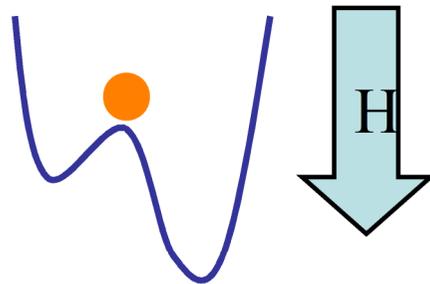


$$H \geq H_c$$



**Dynamics:** Precessional reversal

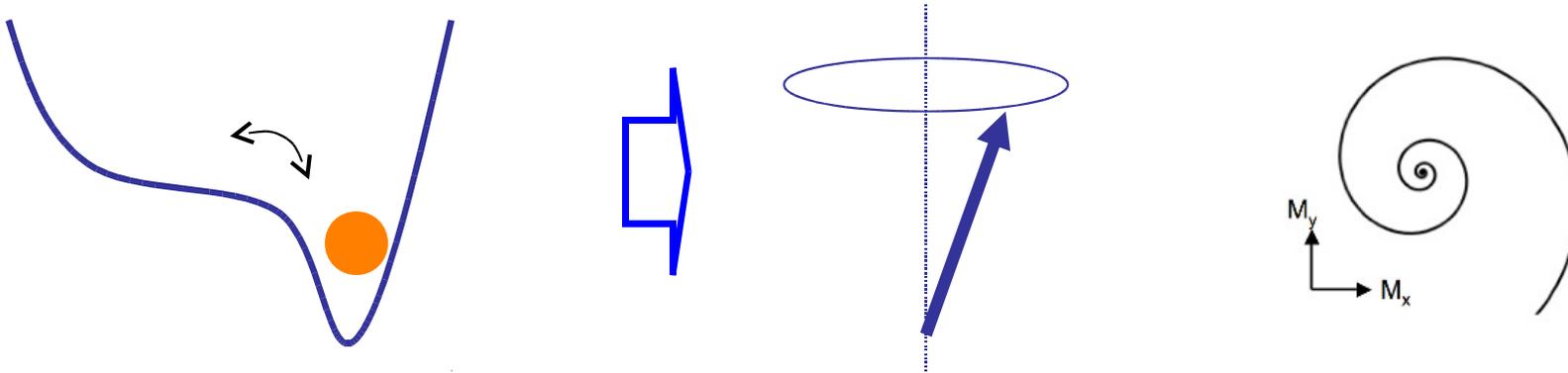
$$H < H_c$$



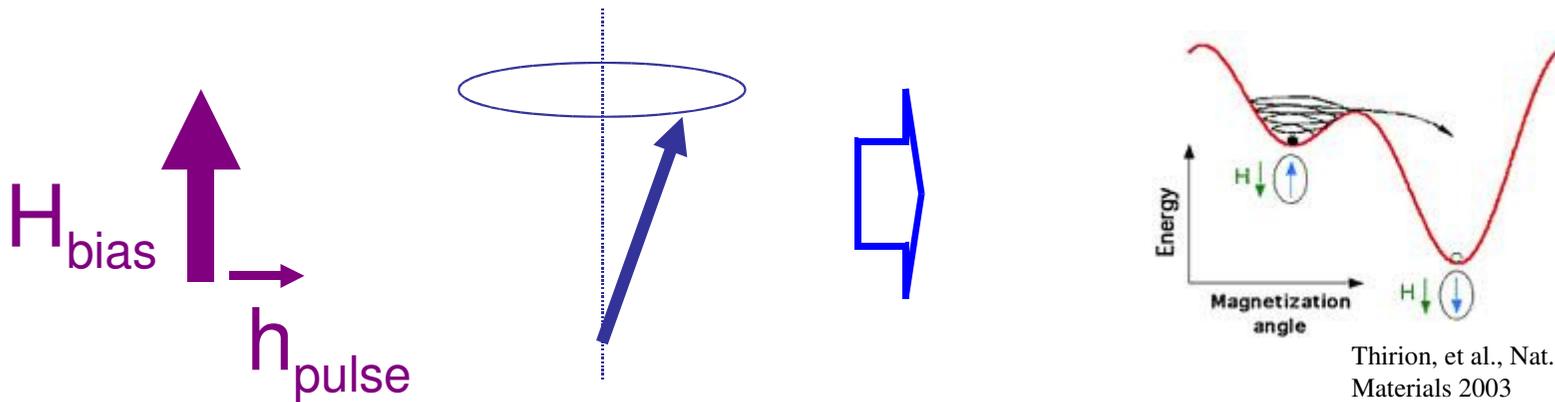
**Stability:** Thermal activation

# Fast Reversal of Single Particles

Switching involves **precession + dissipation**:

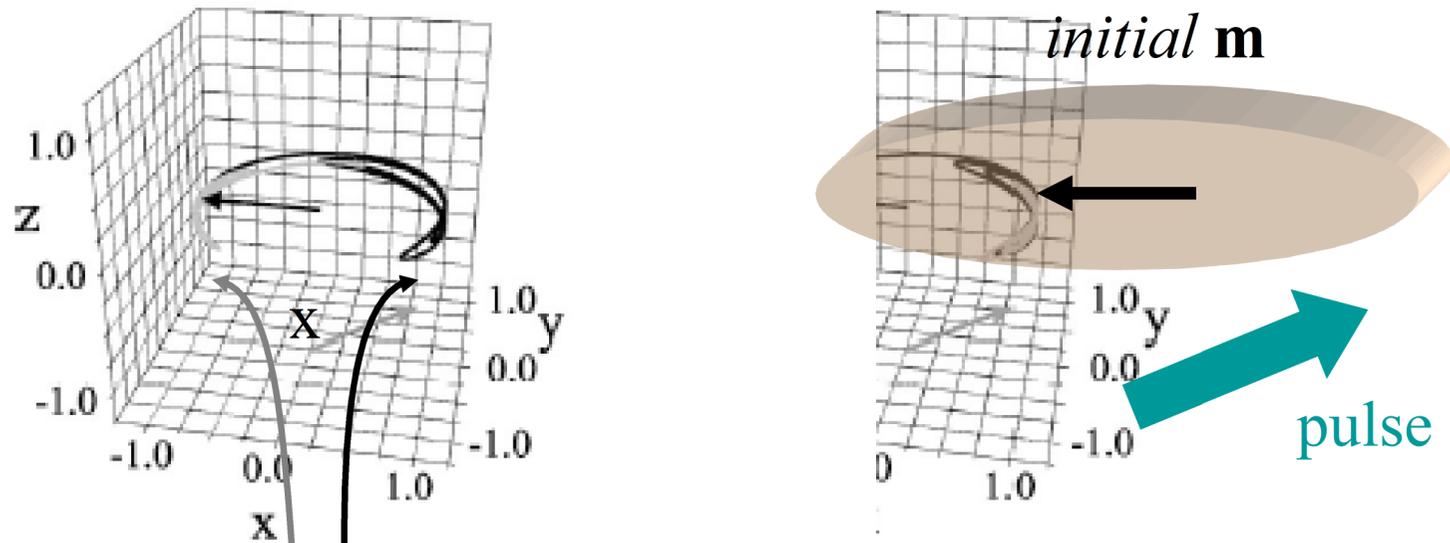


**Experiment:** Apply pulse, drive reversal



# Reversal: Nonlinearities

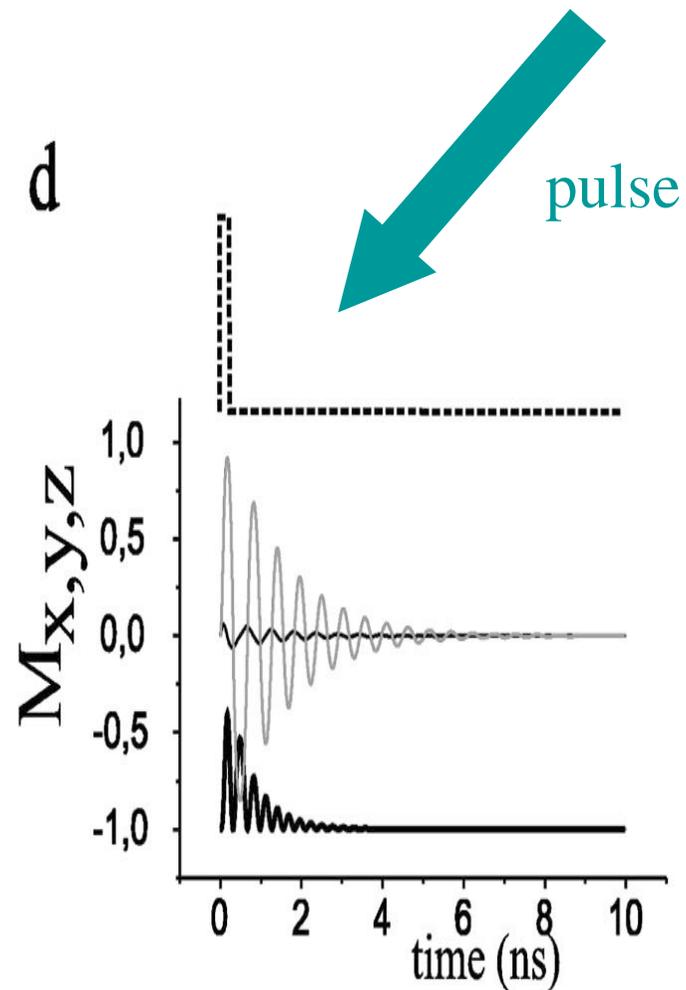
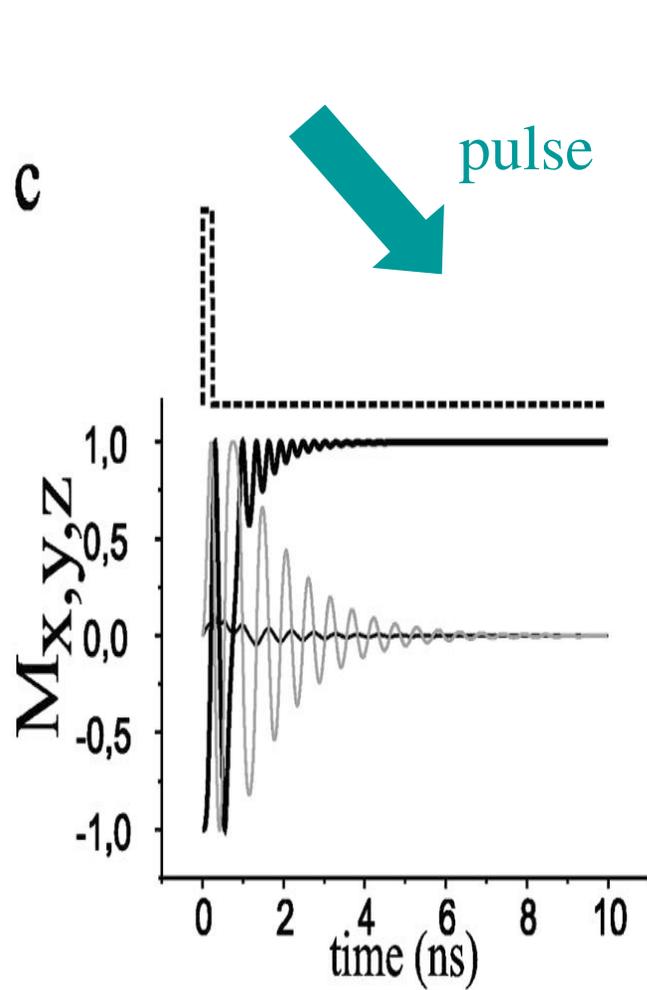
Example with pulse: precession  $\dot{\mathbf{m}}$  with uniaxial  $\mathbf{e}$  anisotropy  $\cdot$  first 10 ns



*black trajectory = during pulse*

*gray trajectory = after pulse*

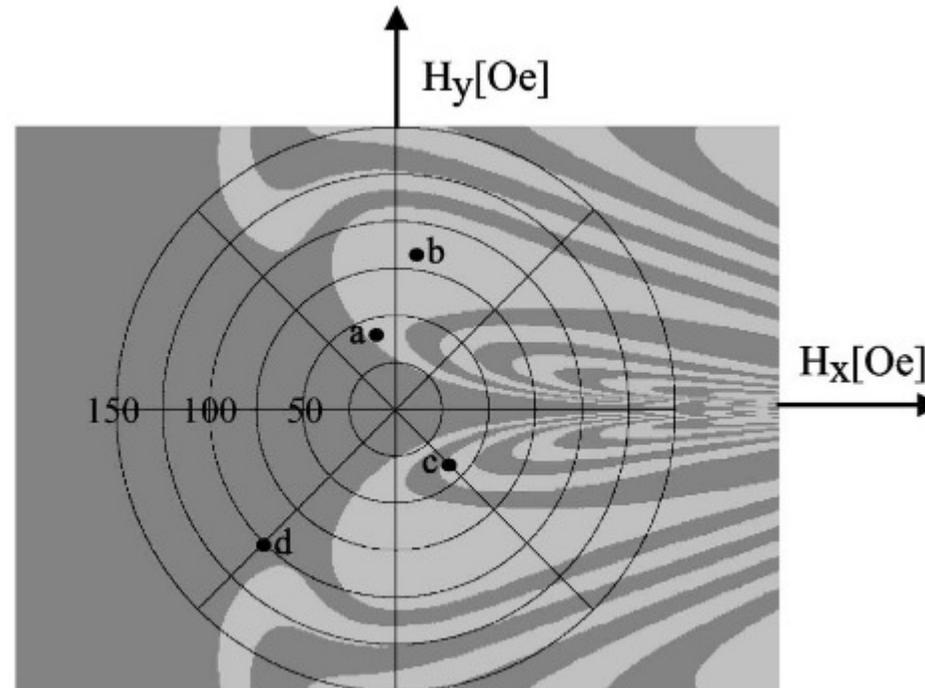
# Sensitivity To Pulse Orientation



# Reversal: Nonlinear Dynamics

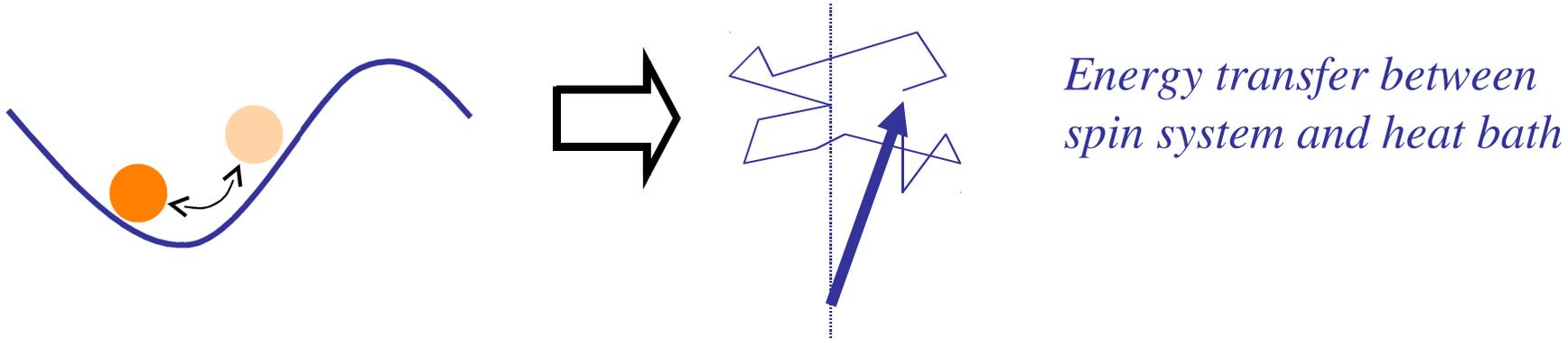
Anisotropy creates windows for precessional reversal

- Pulse: 0.25 ns
- Ellipsoidal particle
- **Polar plot:** field pulse orientation and strength
- **Bright** = switched
- **Dark** = not switched



# Thermal Activation: $H < H_c$

**Climbing to the top: fluctuations**



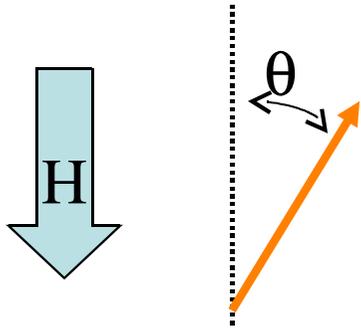
**Torque equation of motion: thermal fluctuation ‘field’**

$$\frac{\partial}{\partial t} \mathbf{m}(\mathbf{r}) = \gamma \mathbf{m}(\mathbf{r}) \times \mathbf{H}_{eff} - \alpha \mathbf{m}(\mathbf{r}) \times \frac{\partial \mathbf{m}(\mathbf{r})}{\partial t} + \mathbf{h}_f$$

*random thermal ‘driving torque’*

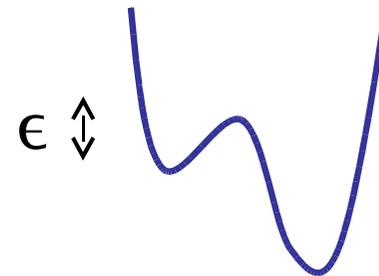
# Single Particle Switching: Stoner-Wohlfarth Model

**Approximate reversal as pure relaxation:**



$$E = V (-H M \cos \theta + K \sin^2 \theta)$$

$$\epsilon = V K + \left[ \frac{H M}{2K} \right]^2$$

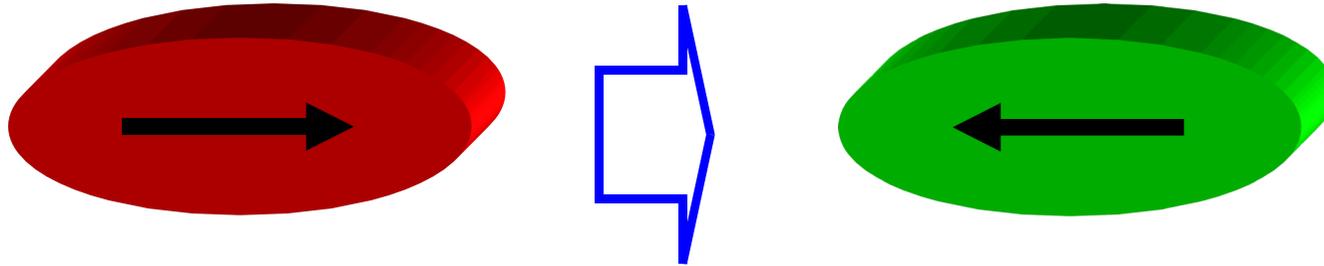


Rate depends on **activation energy** and **attempt frequency**

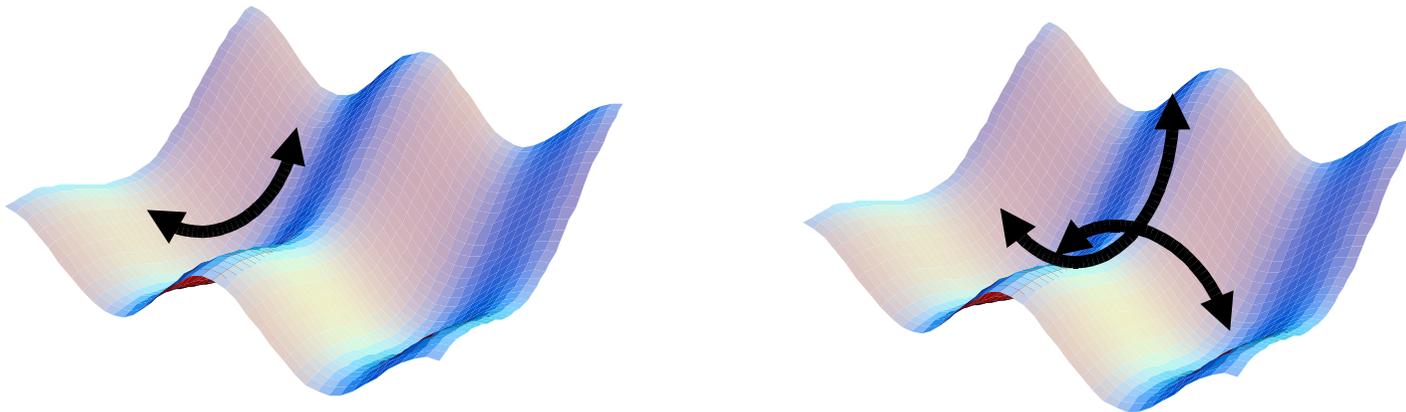
$$\Gamma = \frac{1}{\tau} = f_o \exp(-\epsilon / k_B T)$$

[Kramers, 1940; Langer, 1969; Büttiker, 1981]

# Switching in Elements



**Attempt frequency**  $\sim$  energy landscape **curvatures**

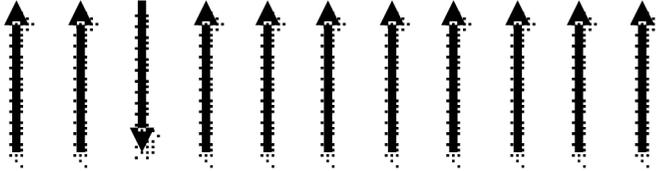


# Spin Waves



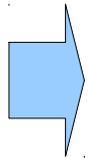
# Fluctuations in Magnetic Density

Energy to reverse one spin in chain:  $2J$

$$H = \sum_{ij} J_{ij} \mathbf{s}_i \cdot \mathbf{s}_j$$


Superposition of ways to flip one spin:

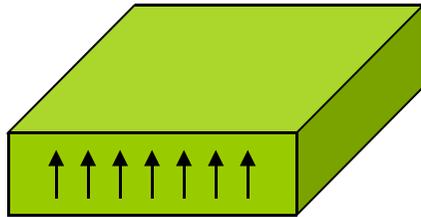
$$|n=1\rangle = |\uparrow\downarrow\uparrow\uparrow\uparrow\rangle + |\uparrow\uparrow\downarrow\uparrow\uparrow\rangle + |\uparrow\uparrow\uparrow\downarrow\uparrow\rangle + \dots$$



*Spin wave excitation (boson)*

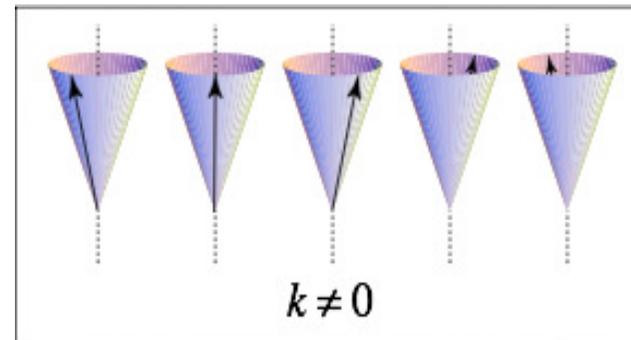
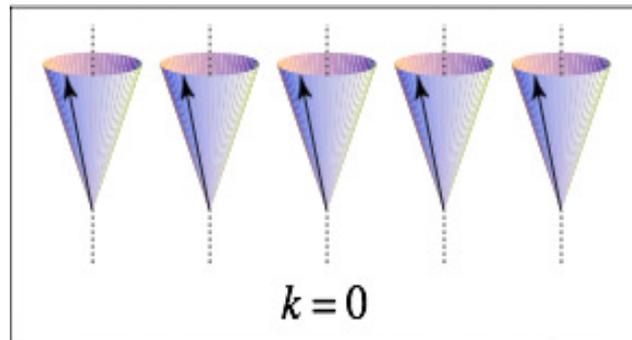
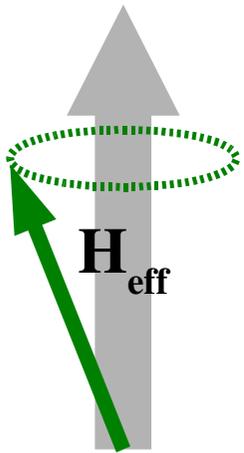
# Classical Picture: Correlated Precession

Ground state magnetic orderings:



- 1) *Magnetic moments*
- 2) *Exchange coupling*

Excitations: Precession dynamics



slide courtesy J-V Kim

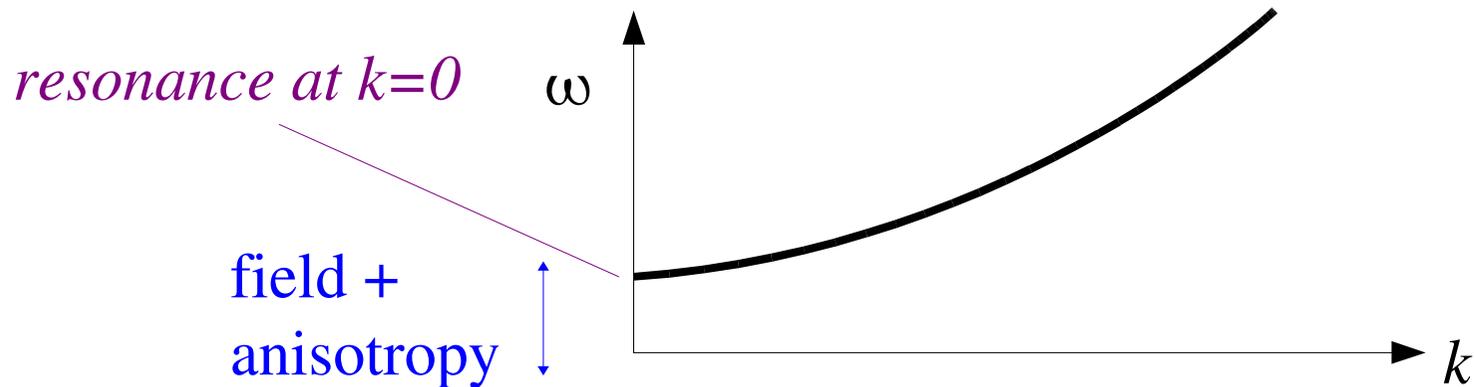
# Dispersion: Spinwaves

Contribution from **exchange**:

$$(m_x, m_y) \sim \exp(-i(\omega t - \mathbf{k} \cdot \mathbf{r}))$$

$$H_{ex} \sim A \nabla^2 m(\mathbf{r}) \quad \Rightarrow \quad H_{ex} \sim -A k^2 m(\mathbf{r})$$

$$\frac{\omega^2}{\gamma^2} = (H_a + A k^2)(H_a - 2 K M_s + A k^2)$$



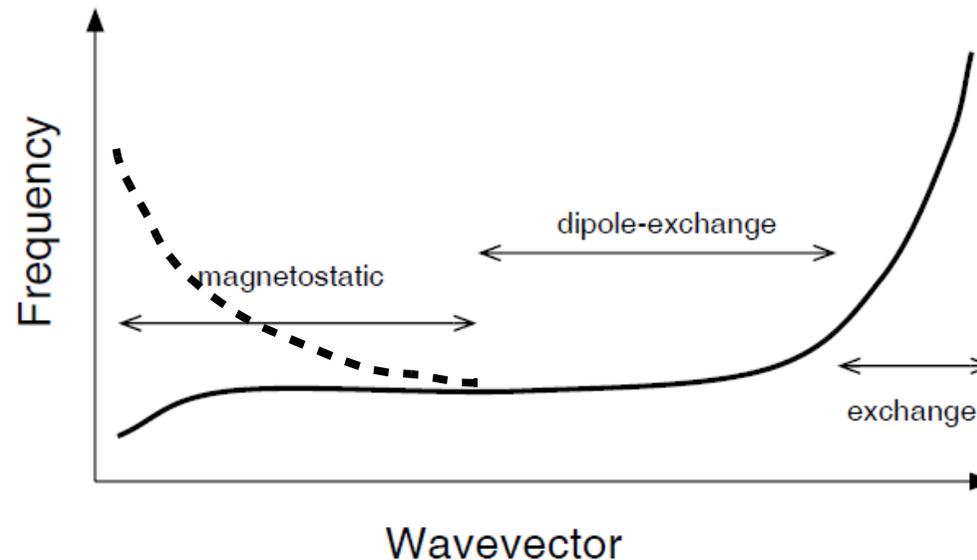
# Dispersion: Dipole Exchange Modes

**Dipolar:** long range nonlocal term

$$h_{dip}(\mathbf{r}) = \int \mathbf{g}(\mathbf{r} - \mathbf{r}') \mathbf{m}(\mathbf{r}') d\mathbf{r}'$$

**Shape anisotropies:** create effective K terms

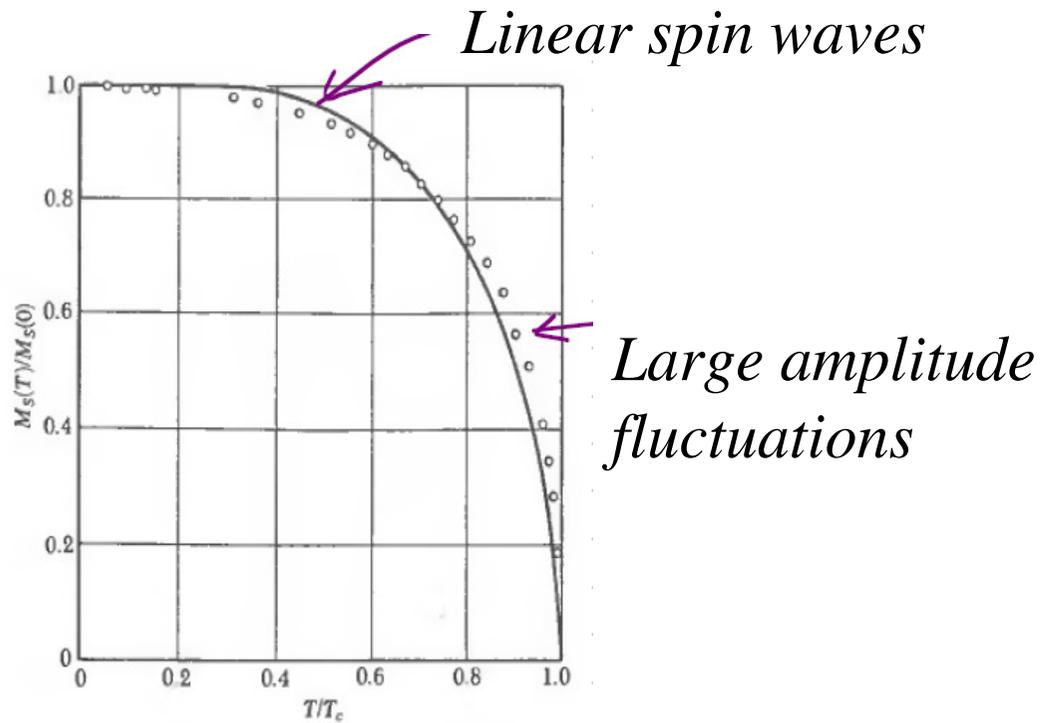
Dynamics:  
**compete with  
exchange** at long  
wavelengths



# Thermal Reduction of Magnetisation



$$M_s(0) - M_s(T) \sim \sum_k \langle n_k \rangle \sim T^{3/2}$$



[Weiss & Forrer]

# Summary II

**'Macroscopic' models** of magnetic configurations & dynamics

**Effective fields:** magnetic parameters

**Reversal processes:** Thermal activation and precession

**Spin waves:** Fluctuations of magnetisation density

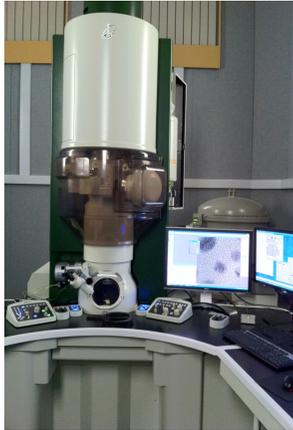


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The End

# Kelvin Nano- characterisation Centre

# JEOL Atomic Resolution Microscope



Standard mode: < 100 pm

Lorentz mode: < 2 nm

*Individual grain Lorentz imaging!*

*Resolve details of vortex structure:*

Field of view: 1 micron (top), 100 nm (bottom)

